INFLUENCE OF THE TEMPERATURE OF A FLOW ON THE READINGS OF A HOT-WIRE ANEMOMETER

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The error due to the flow temperature and sensor wire temperature in measuring the mean velocity of a flow using a hot-wire anemometer is studied.

When measuring the average velocity of a flow using a constant-temperature hot-wire anemometer, the following relationship between the voltage drop along the sensor wire of the hot-wire anemometer to the velocity is usually used:

$$E^2 = A' + B'u^n, \tag{1}$$

where the constants A' and B' are determined during the calibration process. In order to obtain the best precise velocity values, it is necessary that the temperature of the flow T_e at the time of measurement be the same as during the calibration experiments. However, it is difficult to satisfy this condition in practice, since the calibration and measurements are, as a rule, carried out at different times, when the temperature of the flow may be different. Moreover, in many wind tunnels, the value of T_e may even vary during the measurement process. Under these conditions, the coefficients A' and B' in equation (1) will be functions of the temperature T_e. Calibrating the hot-wire anemometer sensor for the various values of the temperature T_e which may occur in an experiment is extremely difficult and requires large expenditures of time. It would be much easier to use a calibration relation of the form (1) obtained for an arbitrary value of T_e provided that coefficients A' and B' are known functions of T_e.

Equation (1) is a special case of a more general relation for the heat exchange between the sensor wire of a hot-wire anemometer and a gas or liquid flow:

$$\mathrm{Nu} = A + B\,\mathrm{Re}^n, \tag{2}$$

where

$$\mathrm{Nu} = \frac{E^2 \alpha R_0}{\pi \lambda l R_w (R_w - R_e)}; \ \mathrm{Re} = \frac{\rho u d}{\mu}.$$

As was shown in [1], there is a large spread in the values of the constants A, B, and n in (2) experimentally obtained by various researchers. There is also no common point of view about what temperature the values of λ, ρ, and μ occurring in the expressions for Nu and Re refer to, so that it would be possible to use equation (2) for various sensor wire temperatures T_w and flow temperatures T_e. There have been attempts to use T_e and T_w, as well as the skin temperature $T_m = 0.5(T_w + T_e)$ (by analogy with the concept of characteristic temperature introduced in [2, 3]) as a defining temperature. Also, in several papers (see, for example, [4]), an additional factor containing the ratio T_w/T_e or T_m/T_e is introduced.

At present, the Collis–Williams relation [4] for an infinite heated wire around which a stream of air flows has come to be the most widely used in experimental research:

$$\mathrm{Nu}_m \left(\frac{T_m}{T_e}\right)^{-0.17} = A_1 + B_1 \mathrm{Re}_m^{0.45}, \tag{3}$$

where $A_1 = 0.24$ and $B_1 = 0.56$. The subscript m indicates that the values of λ, ρ, and μ are those appropriate to the skin temperature T_m. In this case, it is assumed that (3) is valid for any values of T_w and T_e.

It is more convenient to use the following calibration function in place of (1) when analyzing the influence of temperature on the hot-wire anemometer readings:

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$$\frac{E^2}{R_w(R_w-R_e)}=A_2+B_2u^n, \tag{4}$$

where, in agreement with the general relation (2),

$$A_2=A\frac{\pi\lambda l}{\alpha R_0};\quad B_2=B\frac{\pi\lambda l}{\alpha R_0}\left(\frac{\rho d}{\mu}\right)^n. \tag{5}$$

When written in this way, the left-hand side of (4) is more conservative with respect to variations of T_w and T_e.

There are several approaches to evaluating the influence of T_e on the output signal of the hot-wire anemometer [5]. In several papers (for example, [6, 7]), it is assumed that the temperature dependence of the parameters λ, ρ, and μ can be neglected to first approximation; it then follows from (4) that for, say, T_w = const, the departure of the quantity E^2 from its calibration value must be proportional to the corresponding temperature deviation $(T_w - T_e)$. For a more rigorous estimate of the temperature error, the temperature dependence of the parameters λ, ρ, and μ is also taken into account [8, 9, 10]. However, even in this case, the calculated value of the temperature error will depend on the particular form chosen for generalized relation (2). There are no systematic experimental data in the literature which would allow the validity of the existing methods for calculating the temperature error of a hot-wire anemometer to be evaluated. The results for quantitative estimates of the magnitude of this error are highly inconsistent.

In the present work, new experimental results are presented which allow the available information on the question at hand to be improved and allow well-founded practical recommendations for estimating the influence of the flow temperature and the temperature of the hot-wire anemometer sensor wire to be developed on the basis of measured results.

The experiments were carried out in a low-turbulence ($\sqrt{(\overline{u'})^2}/u$ = 0.1-0.2%) wind tunnel with a flow velocities of 1-17 m/sec and flow temperatures from 288 to 323°K. The flow velocity was measured with a DISA 55 A01 constant-temperature hot-wire anemometer. A platinum wire with diameter d = 5.3 μm and length ℓ = 0.965 mm (ℓ/d = 182) was used as the sensor of the hot-wire anemometer. The distance between the wire holders, around which the flow moved longitudinally, was 2 mm.

Control measurements of the flow temperature were carried out using thermocouples with an error of no greater than 0.2°K. The actual flow velocity was determined using a total thrust tube and the statistical pressure recorded on the wall of the tunnel at the cross section where the sensor wire of the hot-wire anemometer was located. The measured pressure drop was recorded with a high-sensitivity manometer whose rms error σ = 0.004 mm of water.

Three series of experiments, corresponding to three different regimes of operation of the hot-wire anemometer, were carried out:

a) T_w = const, T_e = var. This measurement regime corresponds to conditions under which the temperature of the flow varies during the experiment, while the wire temperature T_w remains constant. The experiments were carried out at values of T_w = 450, 500, 550, and 600°K, and the flow temperature was varied from 288 to 323°K in each case;

b) $\Delta T = T_w - T_e$ = const, T_e = var. This case corresponds to conditions under which the overheating of the sensor wire $T_w - T_e$ was held constant independent of the value of T_e. Experiments were carried out for ΔT = 150, 200, 250, and 300°K;

c) $k = R_w/R_e$ = const, T_e = var. This regime corresponds to the most realistic conditions for real measurements, where the temperature T_e is held constant during the experiment, but its value differs from the value T_{ecal} at which the calibration was carried out. Thus, a unique value of T_w was determined for each value of ($R_w = kR_e$). Studies were made for k = 1.4, 1.6, 1.8, and 2.0.

Typical results of measuring the value of E^2 as a function of $u^{0.45}$ (of the form (1)) for various values of T_e in the three measurement regimes are shown in Fig. 1. Clearly, the experimental data can be described well by straight lines, with the coefficients A' and B' depending on the value of T_e. The greatest influence of T_e is observed for T_w = const, and the least for k = const.

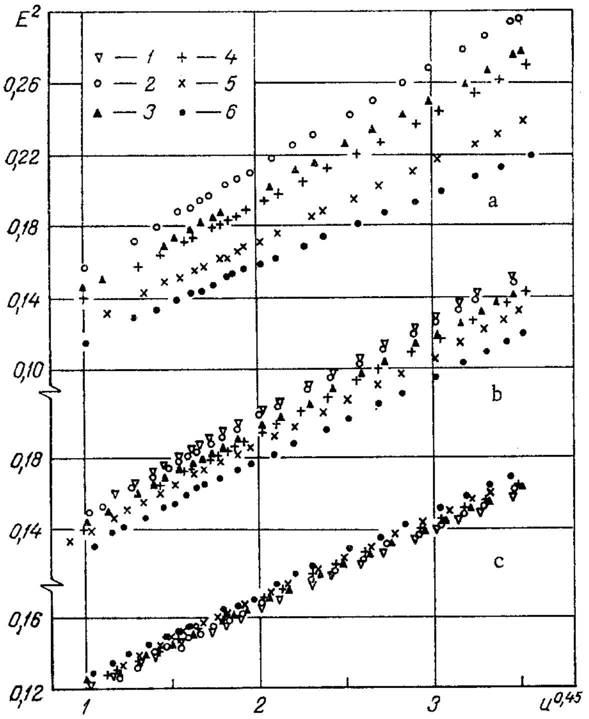

Fig. 1

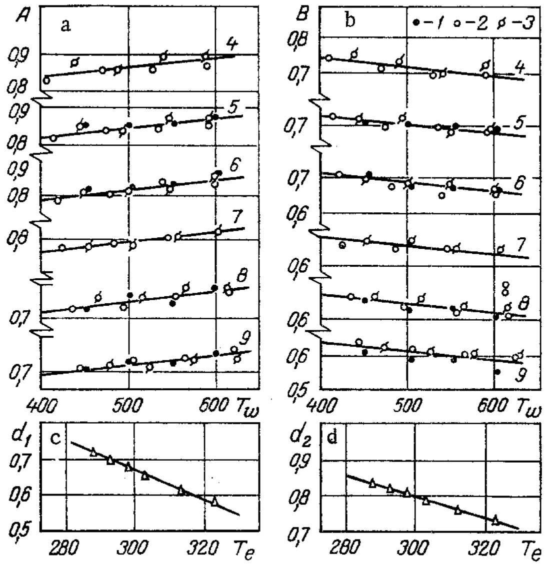

Fig. 2

Fig. 1. Typical results from experimental measurements of the influence of the flow temperature on the calibration relation for a hot-wire anemometer (E, V; u, m/sec). Measurement regimes: a) T_w = 500°K; b) ΔT = 200°K; c) k = 1.6; 1) T_e = 288°K; 2) 293°K; 3) 298°K; 4) 303°K; 5) 313°K; and 6) 323°K.

Fig. 2. Coefficients of calibration relation (6) as a function of T_w(K) and T_e(K). Measurement regimes: 1) T_w = const; 2) ΔT = const; 3) k = const; 4) T_e = 288°K; 5) 293°K; 6) 298°K; 7) 303°K; 8) 313°K; 9) 323°K.

It should be noted that the coefficients A' and B' are dimensional quantities; their quantitative values depend on the geometrical dimensions of the wire and the wire material in a particular sensor. To obtain some more general information, it is appropriate to represent the results of the present study in dimensionless form, which will allow them to be used for sensors with arbitrary parameters. And so, analysis of the experimental data using a generalized form of the calibration relation,

$$\mathrm{Nu}_0 = A + B\,\mathrm{Re}_0^{0.45} \tag{6}$$

(where the subscript "0" indicates that the quantities λ, ρ, and μ occurring in the expressions for Nu and Re are referred to a temperature T_0 = 272°K), was carried out here.

If (6) is rewritten in the form

$$\frac{E^2F_0}{R_w(R_w - R_e)} = A + Bu^{0.45}H_0, \tag{7}$$

where $F_0 = \alpha R_0/\pi\lambda_0\ell$ and $H_0 = (\rho_0 d/\mu_0)^{0.45}$, the coefficients A' and B' in (1) and the dimensionless coefficients A and B in (7) will differ by the following factors:

$$A' = A\frac{R_w(R_w - R_e)}{F_0},\ B' = BR_w(R_w - R_e)\frac{H_0}{F_0}. \tag{8}$$

In the present work, the constants A' and B' were determined for experimental relations of the form (1) (Fig. 1) using the method of least squares. The dimensionless values of A and B determined from (8) for all of the flow regimes studied are shown as a function of T_w in Figs. 2a and b; the quantity T_e is a parameter. Clearly, the coefficients A and B for each value of T_e can be approximated by linear relationships of the form

$$A = c_1T_w + d_1,\quad B = c_2T_w + d_2,$$

and the coefficients c_1 and c_2 can be assumed to be constants independent of T_e within the experimental scatter of the measured points, while the coefficients d_1 and d_2 are linear functions of the temperature T_e (Figs. 2c, d):

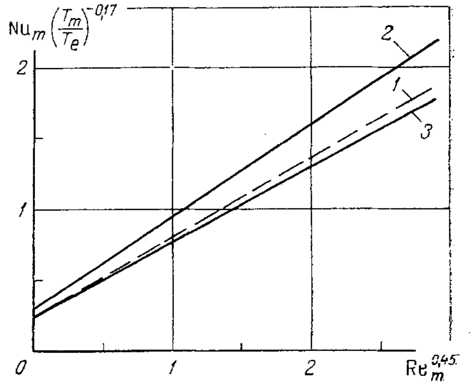

Fig. 3

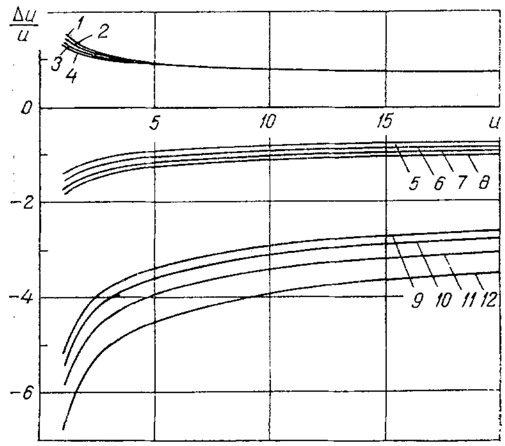

Fig. 4

Fig. 3. Influence of the flow temperature on the calibration relation in coordinates appropriate to the Collis–Williams relations [4]: $\ell/d \to \infty$; T_w = 500°K; 1) calculation using (3); calculation using (6) and (13); 2) T_e = 273°K; 3) 323°K.

Fig. 4. Relative error in the measurement of the flow velocity $\Delta u/u$ (%) for a change of 1°K in the flow temperature as a function of flow velocity u (m/sec). Measurement regimes: a) k = const; 1) k = 2.0; 2) 1.8; 3) 1.6; 4) 1.4; b) ΔT = const; 5) ΔT = 150°K; 6) 200°K; 7) 250°K; 8) 300°K; c) T_w = const; 9) T_w = 600, 10) 550°K; 11) 500°K; 12) 450°K.

$$d_1 = a_1 + b_1T_e,\ d_2 = a_2 + b_2T_e.$$

Hence, we obtain the following expressions for A and B in the general case:

$$A = a_1 + b_1T_e + c_1T_w,\quad B = a_2 + b_2T_e + c_2T_w, \tag{9}$$

from which it follows that the influence of the temperatures T_e and T_w on the values of the coefficients A and B is additive.

Reduction of the experimental data shown in Fig. 2 using the method of least squares allowed the following dependences of the coefficients A and B on T_e and T_w to be obtained for ℓ/d = 182:

$$\begin{aligned} A &= 1.9 - 4.1\cdot10^{-3}T_e + 0.286\cdot10^{-3}T_w,\\ B &= 1.7 - 3.02\cdot10^{-3}T_e - 0.231\cdot10^{-3}T_w. \end{aligned} \tag{10}$$

The rms deviations of the experimental values of A and B from the empirical functions in (10) for the complete experimental data set were σ_A = 1.5% and σ_B = 2%, respectively.

In order to go from the expressions in (10) obtained for a finite-length sensor wire (ℓ/d = 182) to the case $\ell/d \to \infty$, we use the empirical relations for the coefficients of the calibration relation written in the Collis–Williams form (3) from [11]:

$$\left.\begin{aligned} A_1 &= 0.13 + \left(23\frac{d}{d_0} + 79\right)\left(\frac{l}{d}\right)^{-1},\\ B_1 &= 0.5 + \left(25.7\frac{d}{d_0} + 16\right)\left(\frac{l}{d}\right)^{-1} \end{aligned}\right\}\ \text{for}\quad l/d \leqslant 500, \tag{11}$$

$$\left.\begin{aligned} A_1 &= 0.25 + \left(21\frac{d}{d_0} + 22\right)\left(\frac{l}{d}\right)^{-1},\\ B_1 &= 0.53 + \left(24{,}2\frac{d}{d_0} + 8\right)\left(\frac{l}{d}\right)^{-1} \end{aligned}\right\}\ \text{for}\quad l/d > 500, \tag{12}$$

where d_0 = 5.4 μm. It should be noted here that the coefficients A_1 and B_1 depend on both ℓ/d and the wire diameter d. For the experimental data, we have the following from (11) and (12):

$$\frac{A_1\,(l/d\to\infty)}{A_1\,(l/d = 182)} = 0.361,\quad \frac{B_1\,(l/d\to\infty)}{B_1\,(l/d = 182)} = 0.892.$$

Assuming that the influence of the temperatures T_w and T_e on the calibration relation do not depend on the elongation of the wire ℓ/d or its diameter d, and that

$$\frac{A_1(l/d\to\infty)}{A_1(l/d=182)}=\frac{A(l/d\to\infty)}{A(l/d=182)},\quad \frac{B_1(l/d\to\infty)}{B_1(l/d=182)}=\frac{B(l/d\to\infty)}{B(l/d=182)},$$

we obtain the following expressions for A and B for an infinitely long wire:

$$\left.\begin{aligned}A&=0.685-1.48\cdot10^{-3}T_e+0.103\cdot10^{-3}T_w\\ B&=1.52-2.69\cdot10^{-3}T_e-0.206\cdot10^{-3}T_w\end{aligned}\right\}\ \text{for}\ l/d\to\infty. \tag{13}$$

Together with (13), (6) is a generalized calibration relation, in which (unlike the analogous well-known relations) the influence of the temperatures T_w and T_e (including the temperature dependence of the parameters λ, ρ, and μ) occurred only in the coefficients A and B. A comparison of equation (3) with equations (6) and (13) is shown in Fig. 3. It is obvious that using the Collis–Williams coordinates does not completely remove the influence of T_e on the law describing the heat exchange of heat between the wire and the air flow.

Using generalized relation (6), it is easily possible to transform to particular forms of calibration relation (1) and (4) for any particular hot-wire anemometer sensor. In doing so, the correction for the finite length of the wire can be determined if necessary, using equations (11) and (12). Note that the general form (6) can be used in any measurement regime: T_w = const, ΔT = const, k = const, etc.

In order to make a quantitative estimate of the influence of the variations in the flow temperature T_e on the measured value of the flow velocity u, it is convenient to apply the variational method to equation (6). We write (6) in the following form:

$$E^2=(A+B\,\mathrm{Re}_0^{0.45})\,R_w(R_w-R_e)\frac{1}{F_0}=f(u,T_e,T_w), \tag{14}$$

where the coefficients A and B are, in general, given by (9). Then the error in the measurement of the quantity E^2 due to an increase in the temperatures relative to T_e and T_w,

$$\Delta E^2=\frac{\partial f}{\partial T_e}\Delta T_e+\frac{\partial f}{\partial T_w}\Delta T_w. \tag{15}$$

For constant values of T_e and T_w, the differential ΔE^2 increase can be expressed in terms of the differential Δu:

$$\Delta E^2=\frac{\partial f}{\partial u}\Delta u. \tag{16}$$

Setting (15) and (16) equal to one another, we obtain an expression for estimating the relative error in the measurement of the velocity due to small variations in T_e and T_w:

$$\frac{\Delta u}{u}=\frac{u_{\text{meas}}-u}{u}=\frac{1}{0.45B}\left[\left(\frac{b_1-AL}{\mathrm{Re}_0^{0.45}}+b_2-BL\right)\Delta T_e+\left(\frac{c_1+AM}{\mathrm{Re}_0^{0.45}}+c_2+BM\right)\Delta T_w\right], \tag{17}$$

where $L=\alpha/r_e(k-1)$, $M=\alpha(2k-1)/r_w(k-1)$, $r_e=R_e/R_0$, $r_w=R_w/R_0$, and the values of T_e, T_w, and u correspond to the calibration conditions.

Expression (17) is valid for any measurement regime. Equation (17) becomes much simpler in the special cases T_w = const ($\Delta T_w=0$), ΔT = const ($\Delta T_w=\Delta T_e$), and k = const ($\Delta T_w=k\Delta T_e$). When using Eq. (17) in practice, the coefficients A, B, b, and c occurring in it must be corrected for the finite value of ℓ/d for a particular sensor, in accordance with (11) and (12).

In some cases, it is more convenient to define the error Δu with respect to the measured velocity u_{meas}, since the actual velocity u is not known beforehand. In this case, we shall have

$$\frac{\Delta u}{u_{\text{meas}}}=\frac{\Delta u}{u}\,\frac{1}{1+\dfrac{\Delta u}{u}}, \tag{18}$$

where the quantity $\Delta u/u$ is given by Eq. (17).

It should be noted that Eq. (17) was obtained under the assumption that Δu depends linearly on ΔT_e and ΔT_w. In fact, this relationship becomes nonlinear for large values of ΔT_e. Nevertheless, the estimates made above showed that Eq. (17) yields sufficiently accurate results, as long as the absolute value of ΔT_e does not exceed 10°K. As ΔT_e increases further, it becomes necessary to carry out an additional calibration when the value of Δu becomes comparable to u.

The results of calculating the error $\Delta u/u$ for ΔT_e = 1°K for the hot-wire anemometer sensor used in the present experiments (ℓ/d = 182, d = 5.3 μm) are shown in Fig. 4 as a function of the flow velocity for various measurement regimes. It is obvious that the error $\Delta u/u$ may be either positive or negative, with the smallest error being associated with the measurement regime k = const.

It follows from (17) that the degree of influence of the flow temperature T_e on the error in the measurement of the averaged velocity in general depends on the flow velocity, the diameter and length of the sensor wire of the hot-wire anemometer, the overheating coefficient k and the temperature level T_e.

NOTATION

E, voltage drop along the sensor wire; u, flow velocity; $\sqrt{(\overline{u'})^2}$, rms value of the fluctuations in the longitudinal velocity; A, B, coefficients in the calibration relation; n, exponent in the calibration relation; $Re = \rho ud/\mu$, Reynolds number; λ, μ, and ρ, coefficient of thermal conductivity, coefficient of dynamic viscosity, and density of air, respectively; T, temperature, °K; d, ℓ, diameter and length of sensor wire; $R_w = R_0[1 + \alpha(T_w - T_0)]$, electrical resistance of the heated filament; $k = R_w/R_e$, overheating coefficient of the sensor wire; α, temperature coefficient of resistance of the wire material. Subscripts: e, w, conditions at the flow temperature and the temperature of the heated wire; m, conditions at the skin temperature; 0, conditions for T_e = 273°K.

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